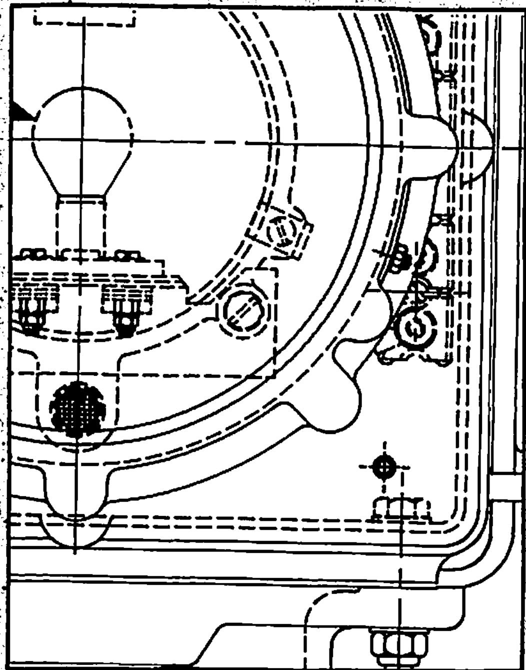


The major kinds of standard gears are explained in this chapter along with the terms and formulas associated with gears. The major gear types included are: spur gears, pinion gears, helical gears, ring gears, bevel gears, worm gears, and rack and pinion gears. The study of gear ratios and gear chains is covered in order to give the beginning designer the required information to achieve the required ratios.

CHAPTER SIXTEEN
Gears

Gears transmit or transfer rotary motion from one shaft to another shaft. Gears can change the direction of rotation, speed up or slow down rotation, increase or reduce power, and change rotary motion into a reciprocating motion. There are various kinds of gears, each with their own function, Figure 16-1.

The drafter must be able to identify each kind of gear, know the various functions of each and be able to design and draw the various gears using correct terminology associated with gears.

Kinds of Gears

Gears are usually classified by the position or location of the shafts they connect. A spur gear, pinion gear or helical gear is usually used to connect shafts that are parallel to each other. Intersecting shafts at 90° are usually connected by a beveled gear or angle gear. Shafts that are not parallel to each other and that do not intersect use a worm and worm gear. In order to connect rotary motion into a reciprocating or back and forth motion, a rack gear and pinion would be used.

Spur Gear

The *spur gear* is the most commonly used gear, Figure 16-2. It is cylindrical in form, with teeth that are cut straight across the face of the gear. All teeth are parallel to the axis of the shaft. The spur gear is usually considered the *driven gear.*

Pinion Gear

The *pinion gear* is exactly like a spur gear but it is usually smaller, and has fewer teeth, Figure 16-2. The pinion gear is normally considered the *drive gear.*

Rack Gear

The *rack gear* is a type of spur gear, but its teeth are in a straight line or flat instead of in a cylindrical form, Figure 16-3. The rack gear is used to transfer circular motion into straight-line motion.

Ring Gear

The *ring gear* is similar to the spur, pinion, and rack gears, except that the teeth are internal, Figure 16-4.

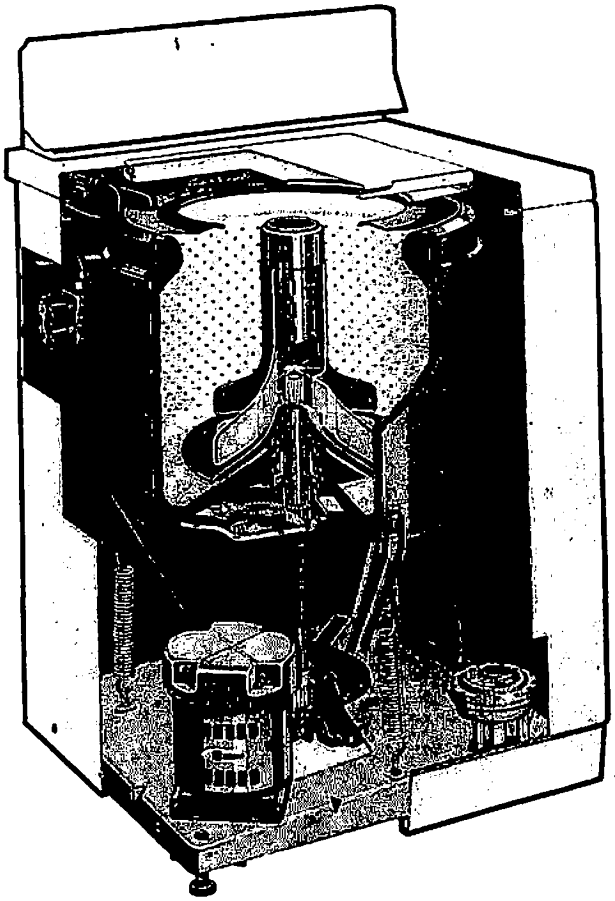


Figure 16-1 Example of gears in use (*Courtesy The Maytag Company*)

Bevel Gear

A bevel gear is another gear commonly used, Figure 16-5. A *bevel gear* is cone shaped in form with straight teeth that are on an angle to the axis of the shaft. Bevel gears are used to transmit power and motion between intersecting shafts that are at 90° to each other.

Angle Gear

The *angle gear* is similar to a bevel gear, except that the angles are at other than 90° to each other.

Miter Gear

The *miter gear* is exactly the same as a bevel gear, except that both mating gears have the same number of teeth. The shafts are at 90° to each other, Figure 16-6.

Spiral Bevel or Miter Gears

Any bevel gears with curved teeth are called *spiral bevel gears*, Figure 16-7.

Worm Gear

Worm gears are used to transmit power and motion at a 90° angle between nonintersecting shafts, Figure 16-8. They are normally used as a speed reducer.

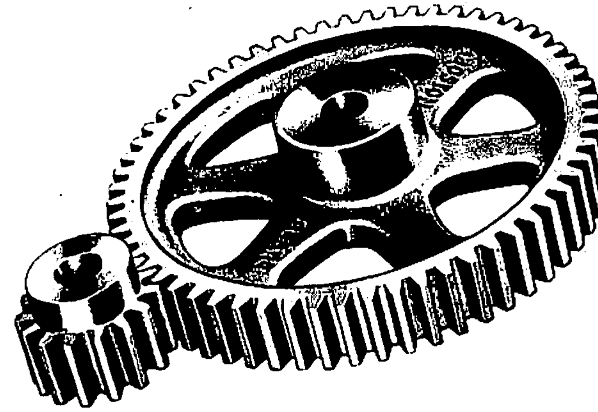


Figure 16-2 Spur gear/pinion gear (*Courtesy Boston Gear*)

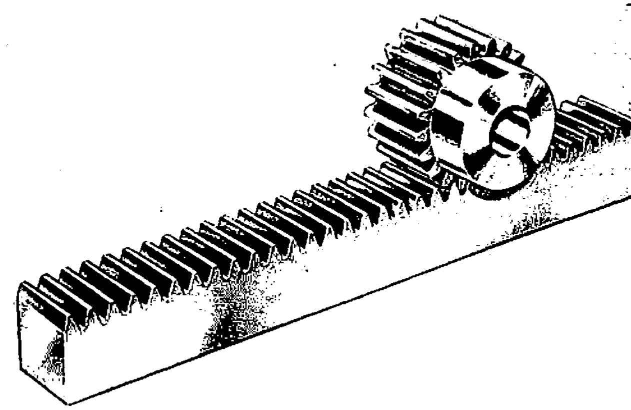


Figure 16-3 Rack gear (*Courtesy Boston Gear*)

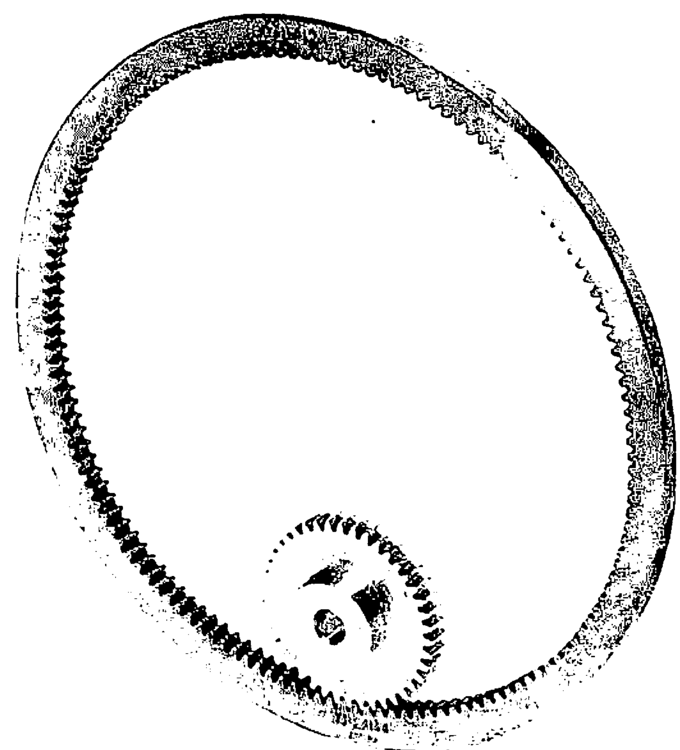


Figure 16-4 Ring gear (*Courtesy Boston Gear*)

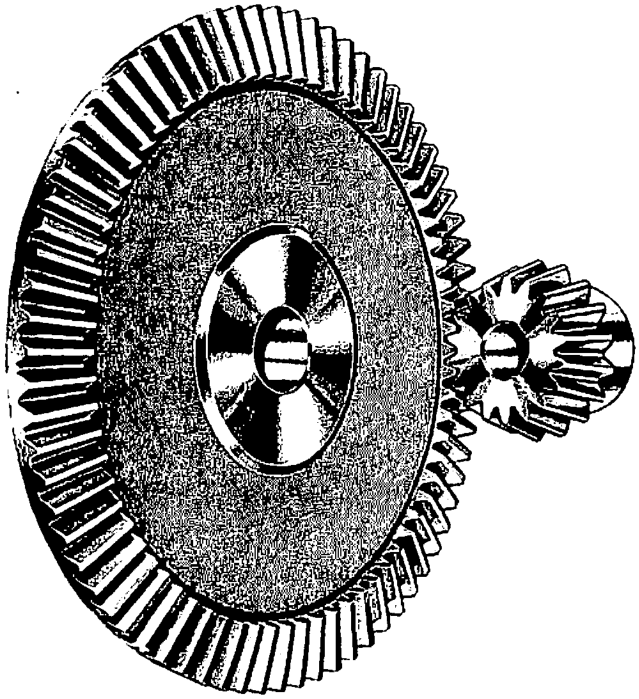


Figure 16-5 Bevel gear (*Courtesy Boston Gear*)

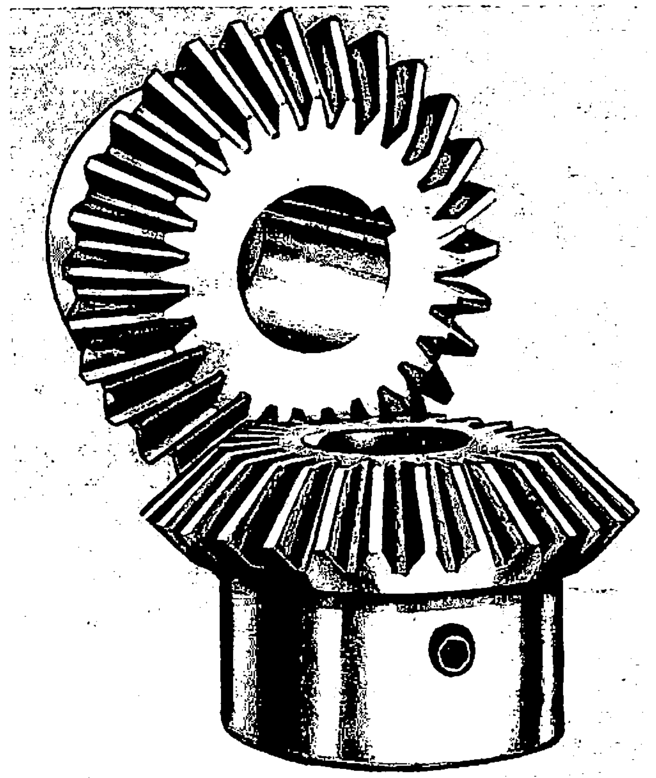


Figure 16-6 Miter gear (*Courtesy Boston Gear*)

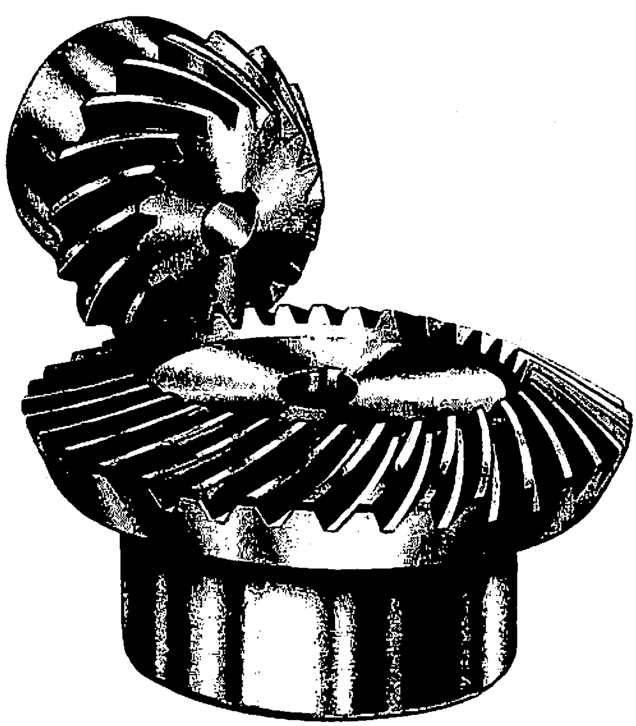


Figure 16-7 Spiral bevel gear (*Courtesy Boston Gear*)

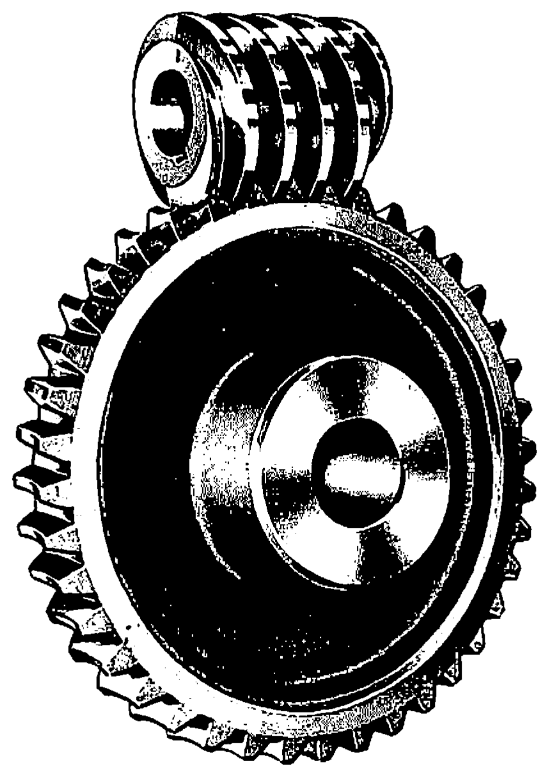


Figure 16-8 Worm gear (*Courtesy Boston Gear*)

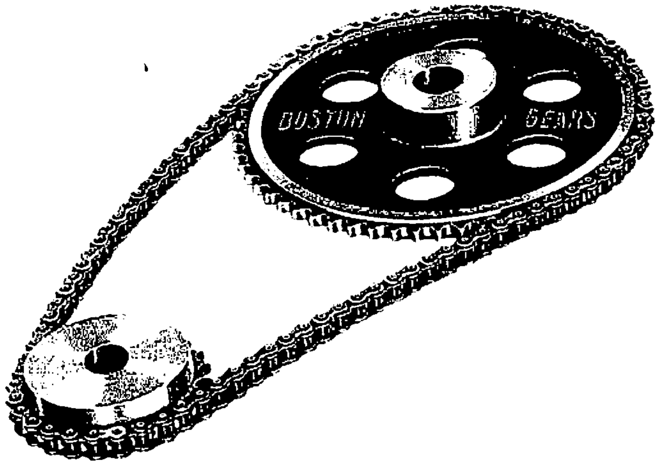


Figure 16-9 Chain and sprockets (*Courtesy Boston Gear*)

The worm gear is round like a wheel. The worm is shaped like a screw, with threads (or teeth) wound around it. Because one full turn of the worm is required to advance the worm gear one tooth, a high-ratio speed reduction is achieved. The worm drives the larger worm gear.

Chain and Sprockets

A *chain and sprockets* are used to transmit motion and power to shafts that are parallel to each other, Figure 16-9. Sprockets are similar to spur and pinion gears, as the teeth are cut straight across the face of the sprocket and are parallel to the shaft. There are many types of chains and sprockets, but all use the same terms, formulas, ratios, and so forth.

Velocity of Feet Per Minute (F.P.M.)

A gear is very similar to a simple wheel. In the field of horology (the science of measuring time), a gear is referred to as a wheel. In making calculations for a gear, it is sometimes easier to think of it as a wheel. Figure 16-10 illustrates the relationship of the diameter of a wheel to given revolutions per minute (R.P.M.) and time. Velocity is calculated by recording the distance that a given point on a wheel or gear travels during a certain period of time. To calculate velocity of a gear, the point is usually assumed to be located on the pitch circle. The pitch circle is discussed in detail later.

The formula is:

$$\pi \text{Dia.} \times \text{R.P.M.} \times \text{Time} = \text{Distance}$$

In this example, point a on the wheel will travel 785 feet.

$$\pi 2.0 \text{ Dia.} \times 500 \text{ R.P.M.} \times 3 \text{ Min.} = \text{Distance}$$
$$6.28 \times 500 \times 3 = 9420 \text{ inches}$$
$$9420 \text{ inches} = 785 \text{ feet}$$

If two wheels of the exact same diameter are placed together, as illustrated in Figure 16-11, and assuming there is no slippage, point a on wheel A will travel the exact same distance as point b on wheel B. Notice, wheel a is rotating clockwise, and wheel B is rotating counterclockwise. If wheel A is rotated at 500 R.P.M. for 3 minutes point a will travel 785 feet.

Gear Ratio

If two friction wheels of different diameters are placed together, Figure 16-12, and assuming there is no slippage, point a on wheel A will travel the same distance as point b on wheel B. In this example, wheel A is the driver.

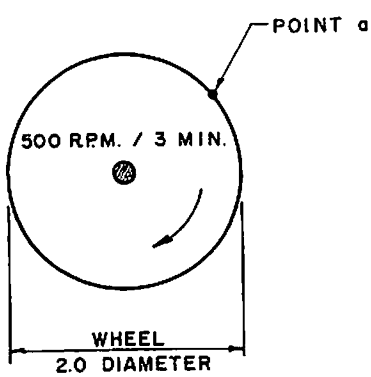


Figure 16-10
Relationship of diameter of a wheel to given revolutions per minute and time

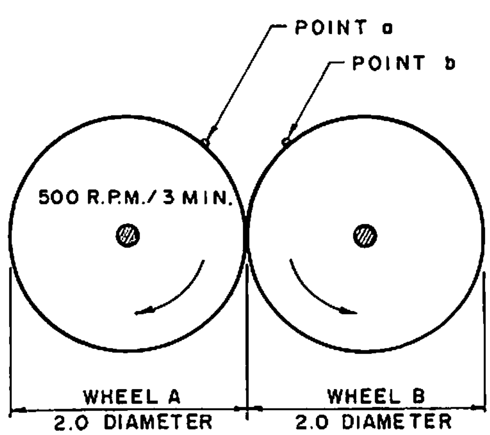


Figure 16-11
Relationship of two wheels of the exact same diameters

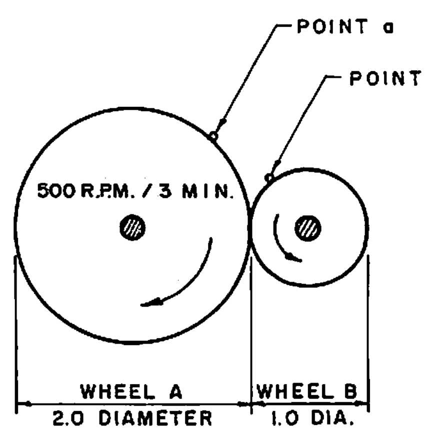


Figure 16-12
Relationship of two wheels of different diameters

π Diameter × revolutions per minute × time = distance

WHEEL A	WHEEL B
πDia. × R.P.M. × time =	πDia. × R.P.M. × time
π2.0 Dia. × 500 R.P.M. × 3 min. =	π1.0 Dia. × ? R.P.M. × 3 min.
6.28 × 500 × 3 = 9420 =	3.14 × 3 = 9.42

9420 ÷ 9.42 = 1000 R.P.M. of Wheel B

Wheel A has a 2.0 diameter and turns at 500 R.P.M. Wheel B has a diameter of half of wheel A, but turns twice as fast, thus point a and point b travel the exact same distance. Note that both wheel A and wheel B travel at the same velocity.

The *ratio* between gears is:

$$\frac{\text{Diameter, wheel A}}{\text{Diameter, wheel B}} = \frac{2}{1} = 2:1$$

The ratio of 2:1 means that wheel B rotates two times each time wheel A rotates once.

The ratio of one gear to another can be calculated by using any of three different methods: the number of teeth of corresponding gears, pitch diameters of corresponding gears, or the R.P.M. between the gears.

Method 1

To find the ratio, divide the number of teeth on the spur gear by the number of teeth on the pinion gear:

$$\frac{\text{Number of teeth (spur gear)}}{\text{Number of teeth (pinion gear)}} = \text{Ratio}$$

Example:

If a spur gear has 60 teeth and a pinion gear has 30 teeth, the ratio would be:

$$\frac{\text{Number of teeth (spur gear)}}{\text{Number of teeth (pinion gear)}} = \frac{60}{30} = 2:1$$

The pinion gear will rotate two times for each full revolution of the spur gear.

Method 2

To find the ratio, divide the pitch diameter (D) of the spur gear by the pitch diameter (D) of the pinion gear:

$$\frac{\text{Pitch Diameter (D) (spur gear)}}{\text{Pitch Diameter (D) (pinion gear)}} = \text{Ratio}$$

Example:

If a spur gear has a pitch diameter (P.D.) of 4.000 and a pinion gear has a pitch diameter (P.D.) of 1.000, the ratio would be:

$$\frac{\text{P.D. (spur gear)}}{\text{P.D. (pinion gear)}} = \frac{4.000}{1.000} = 4:1$$

The pinion gear will rotate four times for each full revolution of the spur gear.

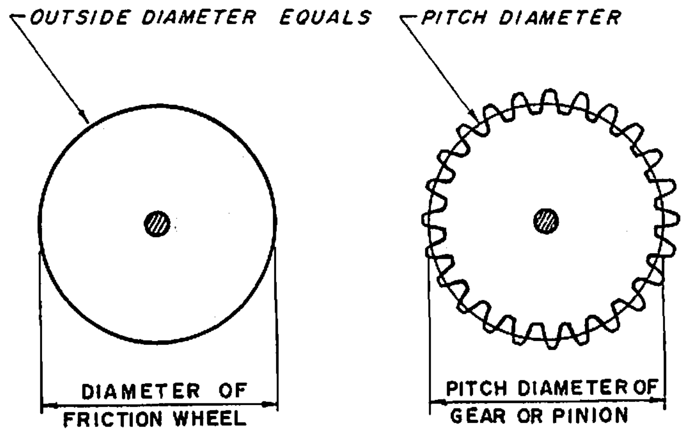


Figure 16-13 Pitch diameter of a gear

Method 3

To find the ratio, divide the revolutions per minute (R.P.M.) of the pinion gear by the R.P.M. of the spur gear:

$$\frac{\text{R.P.M. (pinion gear)}}{\text{R.P.M. (spur gear)}} = \text{Ratio}$$

Example:

If a pinion gear rotates at 1000 R.P.M. and a spur gear rotates at 200 R.P.M., the ratio would be:

$$\frac{\text{R.P.M. (pinion gear)}}{\text{R.P.M. (spur gear)}} = \frac{1000}{200} = 5:1$$

The pinion gear will rotate five times for each full revolution of the spur gear.

Pitch Diameter

Think of the pitch diamter of a gear as the outside diameter of the friction wheel, Figure 6-13. All ratios as discussed so far with the friction wheel apply also to gears. To apply formulas, substitute the outside diameter of the friction wheel for the pitch diameter of the gear.

Gear Blank

Gears are usually cut from a gear blank. The gear blank must allow sufficient space for the gears and a method to attach the gear to the shaft. Illustrated in Figure 16-14 is a common gear blank with an outside diameter and face width for the gears to be cut into, a hub, and a hole for the set screw to secure the gear blank to the shaft. A half-section is used to illustrate the gear blank in this example. Two complete sets of dimensions must be applied to the gear drawing: those relating to the gear blank, as illustrated, and those relating to the teeth. Gear dimensions do not hold tight tolerancing, but all gear tooth dimensions must hold very tight tolerancing.

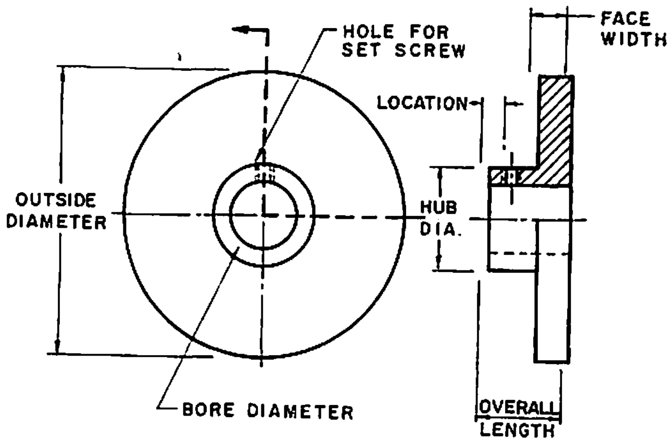


Figure 16-14 Gear blank

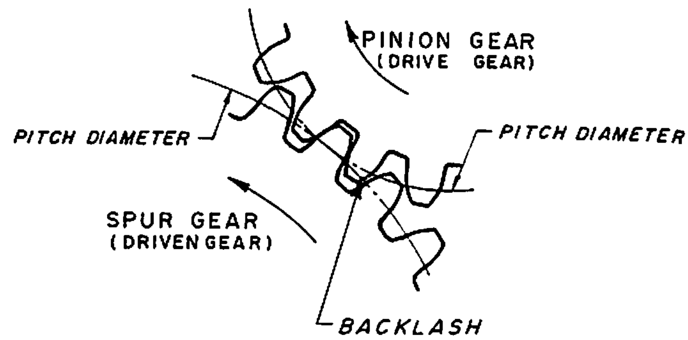


Figure 16-15 Backlash

Backlash

Backlash is the amount by which the gear tooth is less than the tooth space. It is measured by locking one gear and rocking the other back and forth, and measuring that rock at a known radius, usually the pitch diameter. The amount of backlash is the shortest distance between mating teeth as measured between the nondriving surfaces of adjacent teeth, Figure 16-15.

Basic Terminology

The drafter must know and understand all major terms associated with various kinds of gears used today in industry. Illustrated in Figure 16-16 are the basic terms associated with most gears, regardless of type.

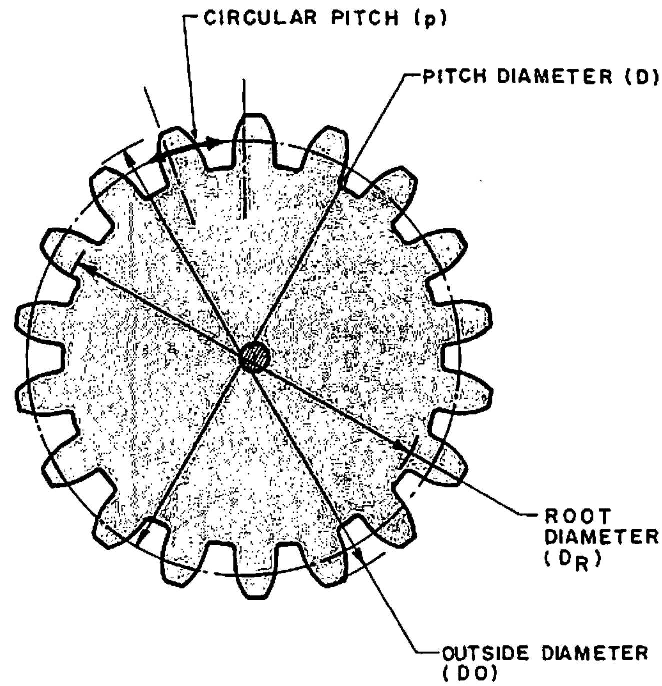


Figure 16-16 Basic gear terminology

Pitch Diameter (D)

The *pitch diameter* is the theoretical circle on which the teeth of mating gear mesh. Think of the pitch diameter on the gear as being the same as the outer diameter on the friction wheel. Note: The pitch diameter is considered the *nominal* size of the gear.

Root Diameter (DR)

The *root diameter* is the measurement over the extreme inner edges of teeth. It is equal to the pitch diameter minus 2 times the dedendum (b).

Outside Diameter (OD)

The *outside diameter* is the measurement over the extreme outer edge of teeth. It is equal to the pitch diameter, plus 2 times the addendum (a).

Circular Pitch (p)

Circular pitch is the distance between corresponding points of adjacent teeth measured on the circumference of the pitch diameter.

Figure 16-17 illustrates the major terms associated with individual gears. There are many more technical terms associated with the actual form and cutting of gear teeth, but the following terms are the ones the drafter should be familiar with.

Working Depth (hk)

The *working depth* is the distance that a tooth projects into the mating space. It is equal to gear addendum (a) plus pinion addendum (a).

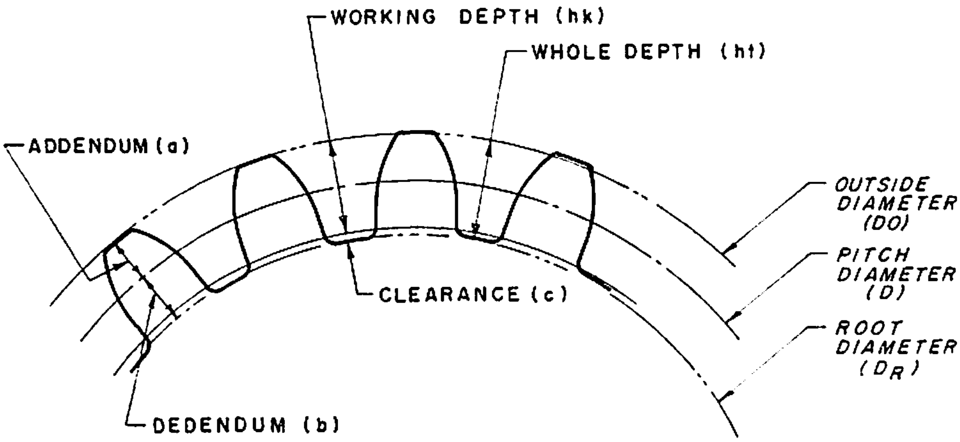


Figure 16-17 Major gear terms

Addendum (a)

The *addendum* is the radial distance from the pitch diameter to the top of the tooth. It is equal to $\frac{1}{P}$.

Dedendum (b)

The *dedendum* is the radial distance from the pitch diameter to the bottom of the tooth. It is equal to $\frac{1.157}{P}$.

Clearance (c)

Clearance is the space between the working depth and the whole depth. It is equal to dedendum (b) minus addendum (a).

Whole Depth (ht)

The *whole depth* is the total depth of a tooth space. It is equal to the addendum (a), plus the dedendum (b), plus the clearance (c).

Note: All notations in parentheses ( ) correspond to those used in the most recent edition of *Machinery's Handbook* so that they agree if a more detailed formula is required by the drafter.

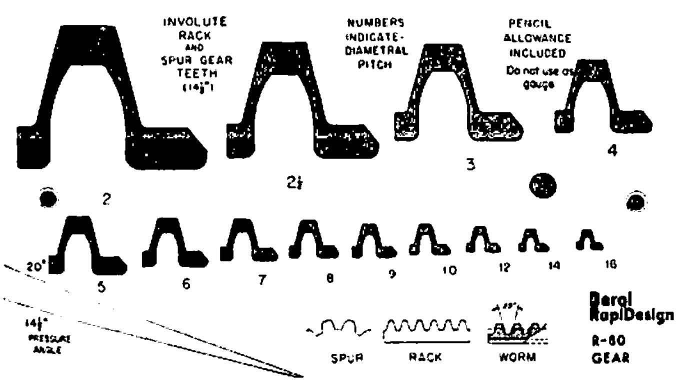


Figure 16-18 Gear template – spur/rack/worm (*Courtesy Modern School Supply*)

Diametral Pitch (P)

Diametral pitch (P) is a ratio equal to the number of teeth on the gear per inch of pitch diameter (D).

$$\text{Diametral pitch (P)} = \frac{\text{Number of teeth (N)}}{\text{Pitch diameter (D)}}$$

Example:

A gear with 48 teeth on a 3.0 pitch diameter (D) would have a diametral pitch (P) of 16.

$$P = \frac{N}{D} = \frac{48}{3.0} = 16$$

Gear Template

A quick and efficient method of illustrating teeth is by using a gear template, Figure 16-18. When using a gear template, the drafter must first calculate the diametral pitch (P). The numbers next to each tooth indicate the diametral pitch. The *lower* part of the template opening is used to draw spur gear *teeth*. The *top* portion of the opening is used to draw teeth of a *rack* or *worm*.

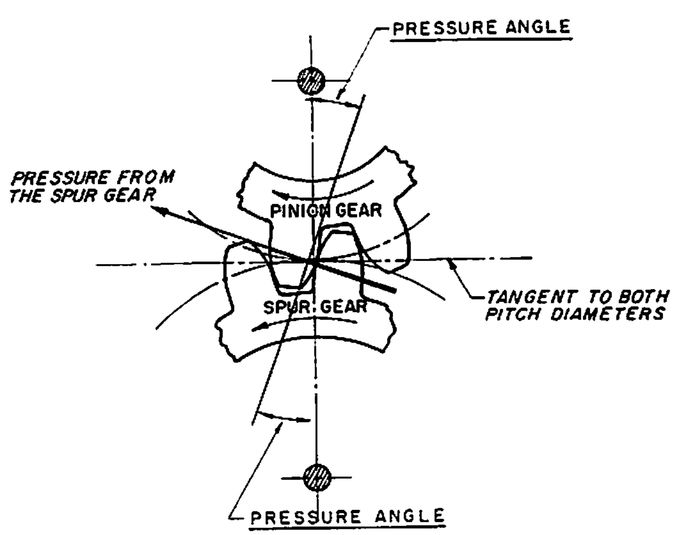


Figure 16-19 Pressure angle

Pressure Angle

The pressure angle determines the tooth form, Figure 16-19. The *pressure angle* is the angle at which pressure from the driver gear tooth passes to the tooth of the driven gear. The standard pressure angle is either 14½° or 20°. The 14½° pressure angle is still popular, especially when replacing old machinery, and because it is quiet. The 20° pressure angle is stronger, due to the heavier tooth section at the base of the tooth form.

Center-to-Center Distances

Calculating the center-to-center distance between two friction wheels is a simple math problem, Figure 16-20.

$$\text{Center-to-center distance} = \frac{\textbf{Outside diameter of wheel A} + \textbf{Outside diameter of wheel B}}{2}$$

In this example, $\frac{2.0 + 1.0}{2} = 1.5$ center-to-center distance

The exact same formula applies to a gear and pinion, except that the pitch diameter (D) is used in place of the outside diameter (OD). Refer to Figure 16-21.

$$\text{Center-to-center distance} = \frac{\textbf{Pitch diameter of gear} + \textbf{Pitch diameter of pinion}}{2}$$

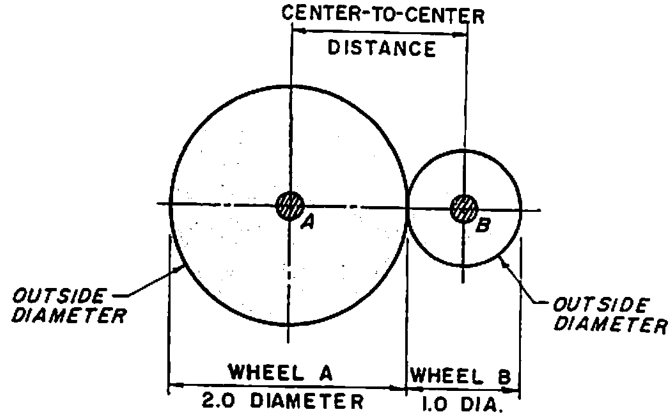


Figure 16-20 Center-to-center distances of two wheels

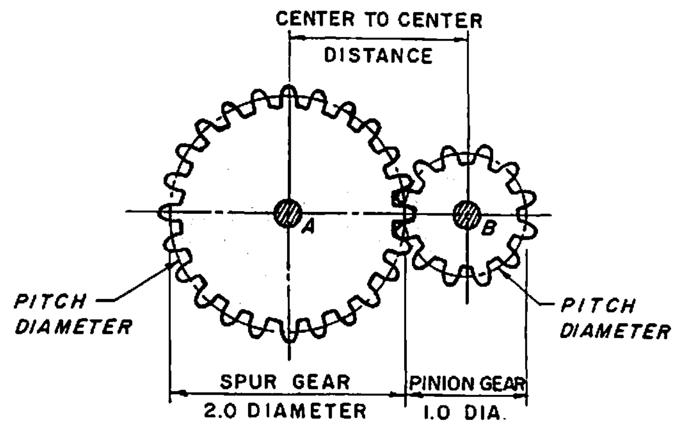


Figure 16-21 Center-to-center distances of two gears

Measurements Required to Use a Gear Tooth Caliper

A *gear tooth caliper* is used to check and measure an individual gear tooth. In order to use a gear tooth caliper, two parts of the individual tooth are used: the chordal thickness and the chordal addendum, Figure 16-22.

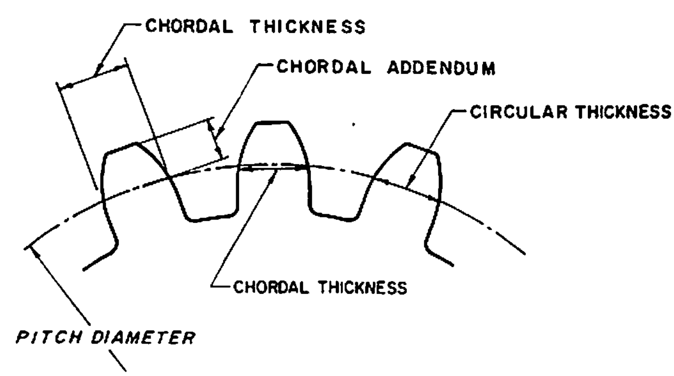


Figure 16-22 Measurements required to use a gear tooth caliper

Chordal Thickness

The *chordal thickness* is the length of the chord measured straight across at the pitch diameter. It is measured straight across tooth, not along the pitch diameter as is the circular thickness.

$$\textbf{Chordal thickness} = \text{Sin}\left(\frac{90°}{N}\right) \times \text{Pitch dia.}$$

Chordal Addendum

The *chordal addendum* is the distance from the top of the tooth to the point at which the chordal thickness is measured.

$$\textbf{Chordal addendum} = \left[1 - \text{Cos}\left(\frac{90°}{N}\right)\right] \times \frac{\text{Pitch dia.}}{2} + \text{addendum}$$

Figures 16-23 and 16-24 show a gear tooth vernier caliper and how it is used.

Required Tooth-Cutting Data

Overall gear blank dimensions are shown on the detail drawing (as illustrated in Figure 16-15) and can

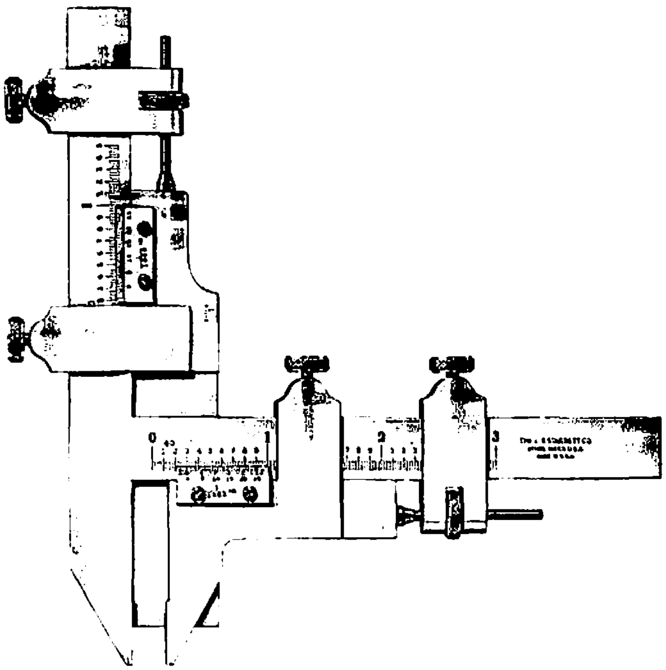


Figure 16-23 Gear tooth caliper (*Courtesy* L. S. *Starrett Co.*)

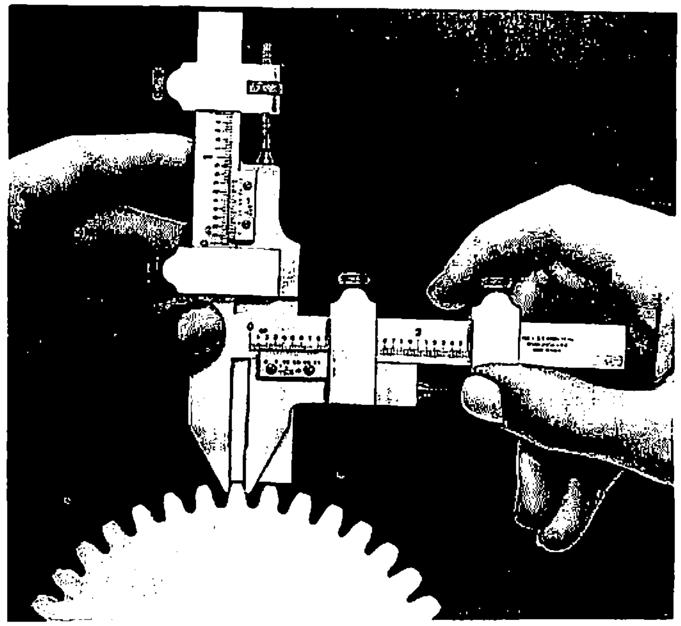


Figure 16-24 Using a gear tooth caliper (*Courtesy* L. S. *Starrett Co.*)

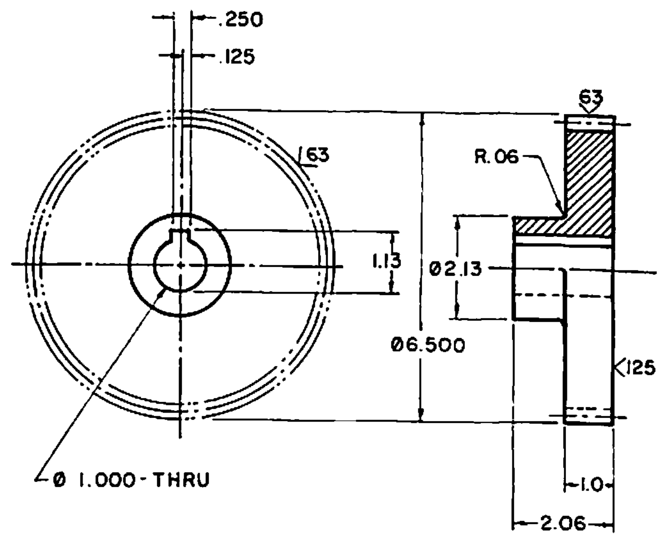
REQUIRED CUTTING DATA			
ITEM	TO FIND:	HAVING	FORMULA
1	Number of teeth (N)	D & P	D x P
		DO & P	(DO x P) - 2
2	Diametral pitch (P)	p	$\frac{3.1416}{p}$
		D & N	$\frac{N}{D}$
		DO & N	$\frac{N+2}{DO}$
3	Pressure angle (Ø)	–	20° STANDARD 14°-30' OLD STANDARD
4	Pitch diameter (D)	N & P	$\frac{N}{P}$
		N & DO	$\frac{N \times DO}{N+2}$
		DO & P	$DO - \frac{2}{P}$
5	Whole depth (ht)	a & b	a + b
		P	$\frac{2.157}{P}$
6	Outside diameter (DO)	N & P	$\frac{N+2}{P}$
		D & P	$D + \frac{2}{P}$
		D & N	$\frac{(N+2) \times D}{N}$
7	Addendum (a)	P	$\frac{1}{P}$
8	Working depth (hk)	a	2 x a
		P	$\frac{2}{P}$
9	Circular thickness (t)	P	$\frac{3.1416}{2 \times P}$
10	Chordal thickness	N,D & a	$\sin\left(\frac{90°}{N}\right) \times D$
11	Chordal addendum	N,D & a	$\left[1 - \cos\left(\frac{90°}{N}\right)\right] \times \frac{D}{2} + a$
12	Dedendum (b)	P	$\frac{1.157}{P}$

Figure 16-25 Required tooth-cutting data for spur and pinion gears

hold loose tolerancing. The actual gear dimensions must hold tight tolerancing.

Twelve essential items of information must be calculated and listed on the gear detail drawing. This list is usually located at the lower right-hand side of the drawing, and generally consists of the data in Figure 16-25, but these items may vary slightly from company to company. The required material, heat-treating process, and other important information must also be included, usually in the title block.

In actual practice, the gear teeth are *not* drawn as it takes too much drawing time. A simplified method is used to illustrate the actual gear tooth, similar to that used to illustrate threads of a fastener, Figure 16-26. The outside diameter and the root diameter are illustrated by a thin phantom line. The pitch diameter is illustrated by a thin center line. Actual teeth are not drawn, except to illustrate some special feature in relationship to a tooth, a spline, a keyway or a locating point. In this example, all dimensions in color are related to the gear blank only. All gear dimensions are called-off in the cutting data box underneath.



CUTTING DATA	
NUMBER OF TEETH	24
DIAMETRAL PITCH	4
PRESSURE ANGLE	20°
PITCH DIAMETER	6.000
WHOLE DEPTH	0.5393
OUTSIDE DIAMETER	6.500
ADDENDUM	.250
WORKING DEPTH	.500
CIRCULAR THICKNESS	.3925
CHORDAL THICKNESS	.2566
CHORDAL ADDENDUM	.3924
DEDENDUM	.289

Figure 16-26 Example of a spur gear detail drawing

Rack

A *rack* is simply a gear with teeth formed on a flat surface, Figure 16-27. A rack changes rotary motion into reciprocating motion. All terms and formulas associated with spur and pinion gears apply to the rack. The sides of the teeth are *straight*, not involute as on a spur gear. The teeth are inclined at the same angle as the pressure angle of the mating pinion gear. Note that the circular pitch of the pinion gear is the same as the linear pitch of the rack. All dimensions for heights of depth, pitch, and addendum lines are calculated from a datum or reference line. The rack is usually manufactured out of rectangular stock but, occasionally, is manufactured out of round stock to meet a specific design.

Two methods can be used to obtain full meshing of the pinion and the rack:

1. The rack is cut down so that it has less backlash.
2. The outside diameter of the pinion is increased slightly so that the pinion is *not* a standard size. If a spur gear is meshed with the oversized pinion gear, its outside diameter is manufactured slightly undersize.

Both methods maintain the standard shaft center-to-center distances between the gear and the pinion. Standard tabulated charts are available to calculate these amounts of increased or decreased outside diameters.

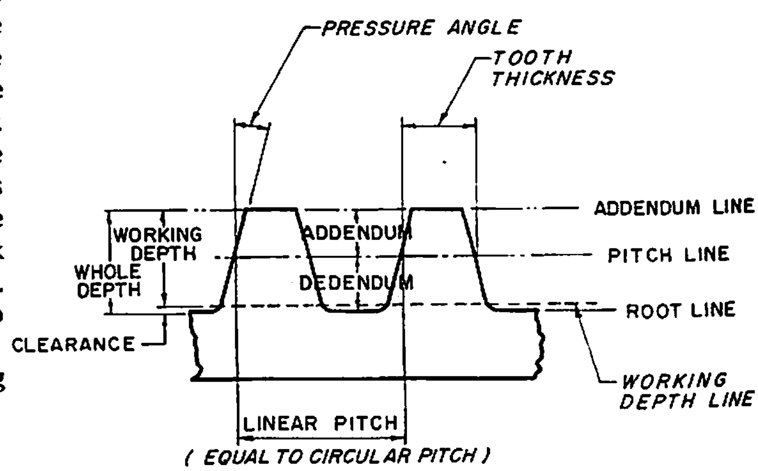
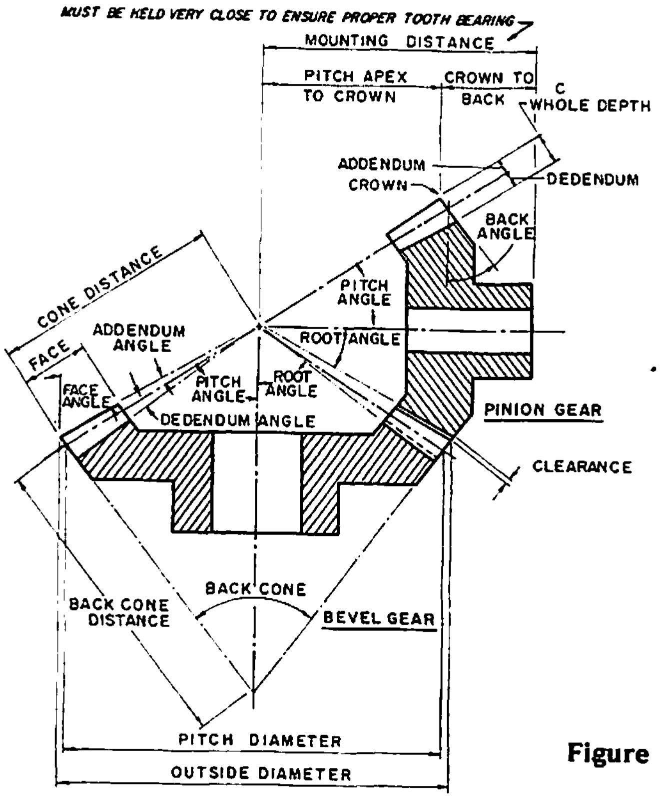


Figure 16-27 Gear terminology for a rack

The required tooth-cutting data for a rack are:

- Pressure angle (same as mating pinion gear)
- Tooth thickness at pitch line (same as mating pinion gear)
- Whole depth (same as mating pinion gear)
- Maximum allowance pitch variation
- Accumulated pitch error max. (over total length)



Bevel Gear

Bevel gears transmit power and motion between intersecting shafts at right angles to each other. Figure 16-28 illustrates the various terms associated with bevel gears. The required tooth-cutting data for bevel gears are similar to spur/pinion gears. These must be calculated and listed on the detail drawing. Figure 16-29 shows the order in which the data and formulas should be listed on the drawing. Bevel gears must be designed and drawn in pairs to ensure a perfect fit.

Worm and Worm Gear

The *worm* and *worm gear* is used to transmit power between nonintersecting shafts at right angles to each other. When using the worm and worm gear, a large speed ratio is possible as one revolution of a single-thread worm turns the worm gear only one tooth and one space.

Figure 16-28 Gear terminology for bevel gears

REQUIRED CUTTING DATA				
ITEM	TO FIND:	HAVING	FORMULA SPUR	FORMULA PINION
1	Number of teeth (N)	–	AS REQ'D.	
2	Diametral pitch (P)	p	$\frac{3.1416}{p}$	
3	Pressure angle (Ø)	–	20° STANDARD 14°-30' OLD STANDARD	
4	Cone distance (A)	D & d	$\sin d \overline{)\frac{D}{2}}$	
5	Pitch distance (D)	p	$\frac{N}{p}$	
6	Circular thickness (t)	p	$\frac{1.5708}{p}$	
7	Pitch angle (d)	N & d (of pinion)	90°-d (pinion)	$\tan d \frac{N\ pinion}{N\ gear}$
8	Root angle (γR)	d & δ	$d - \delta$	
9	Addendum (a)	p	$\frac{1}{p}$	
10	Whole depth (ht)	p	$\frac{2.188}{p} + .002$	
11	Chordal thickness (C)	D & d	$\frac{1}{2}\left(\frac{D}{\cos d}\right)$	$1-\cos\left(\frac{\frac{90°}{N}}{\cos d}\right)+a$
12	Chordal addendum (aC)	d	$\sin\left(\frac{\frac{90°}{N}}{\cos d}\right)$	
13	Dedendum (bC)	P	$\frac{2.188}{P} - a$ (pinion)	$\frac{2.188}{P} - a$ (gear)
14	Outside diameter (DO)	D, a & d	$D + (2 \times a) \times \cos d$	
15	Face	A	1/3 A (max.)	
16	Circular pitch (p)	p & N	$\frac{3.1416 \times p}{N}$	
17	Ratio	N gear & N pinion	$\frac{N\ gear}{N\ pinion}$	
18	Back angle (γO)	–	SAME AS PITCH ANGLE	
19	Angle of shafts	–	90°	
20	Part number of mating gear	–	AS REQ'D.	
21	Dedendum angle δ	A & b	$\frac{b}{A} = \tan \delta$	

Figure 16-29 Required tooth-cutting data for bevel gears

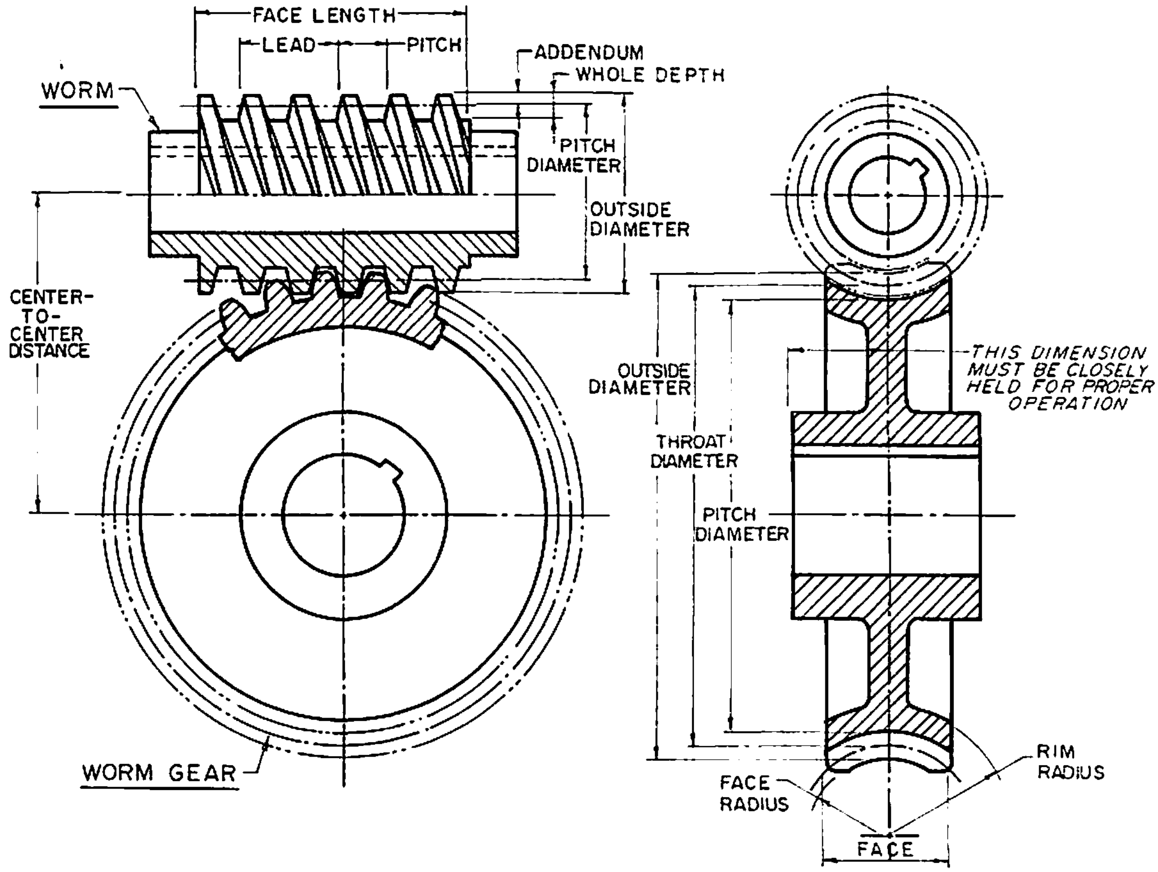


Figure 16-30 Example of worm and worm gear data

The worm's thread is similar in form to a rack tooth. The worm gear is similar in form to a spur gear, except that the teeth are twisted slightly and curved to fit the curvature of the worm.

When drawing the worm and worm gear, an approximate representation is used, Figure 16-30. Cutting data for the worm and worm gear must be listed on the drawing in the lower right side in the order illustrated in Figure 16-31 for the worm gear, and in Figure 16-32 for the worm.

Note: It is important that the mounting of a worm and gear set ensures that the central plane of the gear passes essentially through the axis of the worm. This may be accomplished by adjusting the gear axially at assembly by means of shims. When properly mounted and lubricated, worm gear sets will become more efficient after the initial breaking-in period.

Other information that must be included on a worm/worm gear detail drawing includes:

- Gear blank information
- Tooth-cutting data
- Reference to mating part

Gear Train

A *gear train* is two or more gears used to achieve a designed R.P.M. The ratio of the R.P.M. of the first gear to the R.P.M. of the final gear is called the *value* of the gear train.

Example:

$$\frac{\text{R.P.M. of shaft, first shaft}}{\text{R.P.M. of shaft, final shaft}} = \text{Value of gear train}$$

Study the example on gear trains in Figure 16-33:

- Shafts 1 and 2 are connected by gear A and pinion B
- Shafts 2 and 3 are connected by gear C and pinion D
- Shafts 3 and 4 are connected by gear E and pinion F
- Gear A has 60 teeth
- Pinion B has 15 teeth
- Gear C has 40 teeth
- Pinion D has 20 teeth
- Gear E has 45 teeth
- Pinion F has 9 teeth

Problem:

The R.P.M. of shaft 1 is 200 R.P.M. What is the R.P.M. of shaft 4? What is the gear train value?

Each shaft taken individually:

$$\text{Ratio} = \frac{\text{N spur gear (driver gear)}}{\text{N pinion gear (drive gear)}}$$

R.P.M. of driven pinion = Ratio × R.P.M. driver

REQUIRED CUTTING DATA			
ITEM	TO FIND	HAVING	FORMULA
1	Number of teeth (N)	-	AS REQ'D.
2	Pitch diameter (D)	N & p	$\frac{N \times p}{3.1416}$
3	Addendum (a)	p	p x .3181
		P	$\frac{1}{P}$
4	Whole depth (ht)	p	p x .6866
		P	$\frac{2.157}{P}$
5	Lead (L) Right-Left	p & N	p x N
6	Worm part no.	-	AS REQ'D.
7	Pressure angle Ø	-	20° STANDARD 14°-30' OLD STANDARD
8	Outside diameter (DO)	Dt & Pa	Dt + .4775 x Pa
9	*Circular pitch (p)	P	$\frac{3.1416}{P}$
		L & N	$\frac{L}{N}$
10	Diametral pitch (P)	p	$\frac{3.1416}{p}$
11	Throat diameter (Dt)	D & Pa	D + .636 x Pa
12	Ratio of worm/worm gear	N worm & N worm gear	$\frac{N \text{ worm gear}}{N \text{ gear}}$
13	Center to center distance between worm & worm gear	D worm & D worm gear	$\frac{D \text{ worm} + D \text{ worm gear}}{2}$

Circular pitch (p) must be same as worm Axial pitch (Pa)

Figure 16-31 Required tooth-cutting data for a worm gear

REQUIRED CUTTING DATA			
ITEM	TO FIND	HAVING:	FORMULA
1	Number of teeth (N)	P	$\frac{3.1416}{P}$
2	Pitch diameter (D)	Pa	(2.4 x Pa)+ 1.1
		DO & a	DO - (2 x a)
3	Axial pitch (Pa)	-	Distance from a point on one tooth to same point on next tooth
4	Lead (L) Right or Left	p & N	p x N
5	Lead angle (La)	L & D	$\frac{L}{3.1416 \times D}$ = tan La
6	Pressure angle (Ø)	-	20° STANDARD 14°-30' OLD STANDARD
7	Addendum (a)	p	p x .3183
		P	$\frac{1}{P}$
8	Whole depth (ht)	Pa	.686 x Pa
9	Chordal thickness	N, D & a	$\left[1 - \cos\left(\frac{90°}{N}\right)\right] \times \frac{D}{2} + a$
10	Chordal addendum	N, D & a	$\sin\left(\frac{90°}{N}\right) \times D$
11	Outside diameter (DO)	D & a	D + (2 x a)
12	Worm gear part no.	-	AS REQ'D.

Axial pitch (Pa) must be same as worm gear circular pitch (p)

Figure 16-32 Required tooth-cutting data for a worm

Step 1.

a. Shaft 1 rotates at 200 R.P.M.
b. Gear A has 60 teeth; pinion B has 15 teeth.
c. This ratio is:

$$\frac{\text{N spur gear}}{\text{N pinion gear}} = \frac{60}{15} = \frac{4}{1} \text{ or 4:1 ratio}$$

d. R.P.M. of driven gear (shaft 2) = Ratio × .R.P.M. driver = 4 × 200 = 800 R.P.M.

Step 2.

a. Shaft 2 rotates at 800 R.P.M.
b. Gear C has 40 teeth; pinion D has 20 teeth.
c. This ratio is:

$$\frac{\text{N spur gear}}{\text{N pinion gear}} = \frac{40}{20} = \frac{2}{1} \text{ or 2:1 ratio}$$

d. R.P.M. of driven gear (shaft 3) = Ratio × R.P.M. driver = 2 × 800 = 1600 R.P.M.

Step 3.

a. Shaft 3 rotates at 1600 R.P.M.
b. Gear E has 45 teeth; pinion F has 9 teeth.
c. This ratio is:

$$\frac{\text{N spur gear}}{\text{N pinion gear}} = \frac{45}{9} = \frac{5}{1} \text{ or 5:1 ratio}$$

d. R.P.M. of driven gear (shaft 4) = Ratio × R.P.M. driver = 5 × 1600 = 800 R.P.M.

Step 4.

$$\text{Value of gear train} = \frac{\text{R.P.M. last shaft}}{\text{R.P.M. first shaft}} = \frac{8000}{200} = 40 \text{ value}$$

To calculate the value of a *complete* gear train use the following formula:

The product of the number of teeth on all the drivers (spur gears) divided by the product of the number of teeth on all followers (pinion gears) equals the value of a complete gear train.

Example:

$$\frac{\text{N gear A} \times \text{N gear C} \times \text{N gear E}}{\text{N pinion B} \times \text{N pinion D} \times \text{N pinion F}}$$

$$\frac{60 \times 40 \times 45}{15 \times 20 \times 9} = \frac{108{,}000}{2700} = 40$$

In Figure 16-33, the gear train has been sketched with the gears in a neat row. This is done for the purpose of showing the sketch or basic design. In the final assembly, gears are actually clustered closer together to save space, and they are almost never in a row as shown.

Materials

Gears are made of many materials, such as brass, cast iron, steel, and plastic to mention but a few. Many bevel gears are forged, some are stamped from thin material, some are cast as a blank and machined, and others are die-cast to the exact size and shape.

Each application must be carefully analyzed. Metal gears have been used for years, but, if the load is not excessive, plastic gears have many advantages. Plastic runs quieter, has a self-lubricating effect, weighs much less, and costs much less. In addition, complicated multiple gears can be molded into a single piece which further reduces cost.

Design and Layout of Gears

The initial design of gears starts with the nominal size of the pitch diameters. The required speed ratio, loading, space limitations, and center-to-center distances are also important factors to consider. Mating gears and pinions must have equal diametral pitch in order to correctly mesh; therefore, the diametral pitch should be one of the first considerations. A complete analysis and design of a gear or a complete gear chain are very complex, and far beyond the scope of this text. Most designers try to use standard gears from a company that specializes in gears, gear chains, and gear design. Most gear manufacturing companies can be of assistance in designing gears for special applications. These companies can usually manufacture gears at a lower price than if they were made in-house.

Further analysis, in-depth study, and design data can be found in the most recent edition of *Machinery's Handbook*.

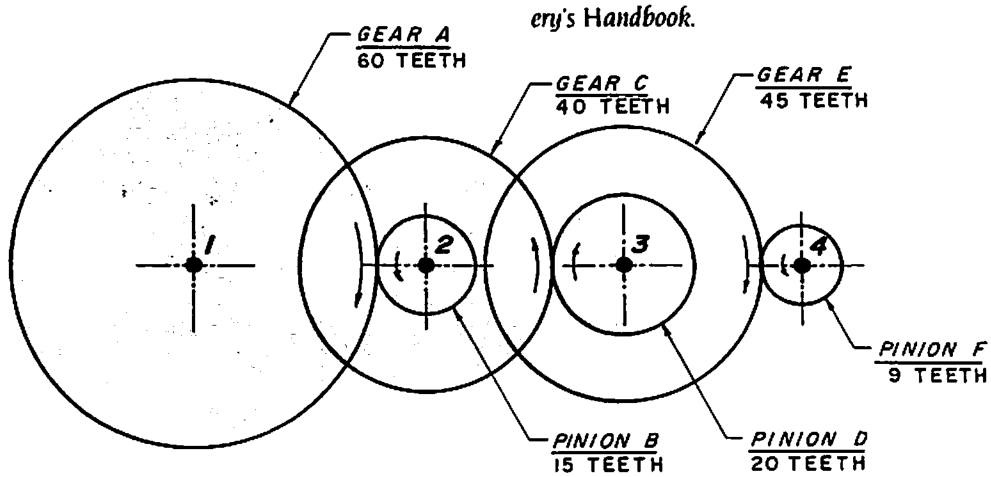


Figure 16-33 Gear train

Review

Calculate the following math problems; include all math work to illustrate how the answers were derived.

1. What is the outside diameter of a spur gear having a pitch diameter of 1.500 and 48 teeth?
2. What is the addendum and dedendum of a spur gear having a pitch diameter of 3.000 and 48 teeth?
3. With the addendum, dedendum, and pitch diameter of Problem 2, what is the root diameter and outside diameter?
4. How many teeth are on a spur gear having a pitch diameter of 1.750, diametral pitch of 20, and an outside diameter of 1.850?
5. What is the pitch diameter of a spur gear having a diametral pitch of 24, 78 teeth, and an outside diameter of 3.333?
6. What is the diametral pitch of a spur gear having 80 teeth and a 2.500 pitch diameter?
7. What is the root diameter of a spur gear having a pitch diameter of 2.000 and 32 teeth?
8. What is the pitch diameter of a spur gear having 40 teeth and a diametral pitch of 20?
9. What is the diametral pitch of a pinion gear having 75 teeth and 3.208 outside diameter? Use the formula $P = \frac{N}{D}$
10. How many teeth are required for a pinion gear with a mating spur gear having a P of 48, a 2.000 pitch diameter, and a 4:1 ratio?
11. What is the whole depth of a spur gear with the following specifications: 70 teeth, .7291 pitch diameter, .750 outside diameter, and 96 diametral pitch?
12. What is the circular pitch of a bevel pinion having a 16 diametral pitch, and 20 teeth?
13. A spur gear has a pitch diameter of 3.000 and a pinion gear has a pitch diameter of 1.500. If an R.P.M. 250 is needed at the shaft of the pinion gear, how fast should the spur gear be rotated?
14. What is the ratio between a spur gear with an R.P.M. of 175 and a pinion gear with an R.P.M. of 1050?
15. The ratio between a pair of bevel gears at a 90° angle and 28 P is:

    Gear: D = .9375, 45 teeth
    Pinion: D = .6250, 30 teeth
16. What is the outside diameter of a bevel gear with the following specifications: a 2.500 pitch diameter, a 48 diametral pitch, a 4:1 ratio, and 120 teeth?
17. What is the shaft center distance between a worm and a worm gear with the following specifications:

Worm:	Pitch diameter .500
	Diametral pitch 24
	Single thread
Worm Gear:	Pitch diameter 4.000
	Diametral pitch 24
	(N) Teeth 100
18. What is the gear ratio of a spur gear and pinion gear with the following specifications? (Calculate the answer using three different methods.)

Spur gear:	Pinion gear:
75 teeth	15 teeth
3.208 DO	.6250 D
3.125 D	.708 DO
725 R.P.M.	24 P
24 P	3/16 face
3/16 face	3625 R.P.M.

Chapter Sixteen Problems

The following problems are intended to give the beginning drafter practice in using the many formulas associated with gears, and practice in designing and laying out finished professional detail drawings of many major kinds of gears.

The steps to follow in laying out gears are:

Step 1. Study the problem carefully.

Step 2. Make a sketch if necessary.

Step 3. Do all math required for each problem. Keep all math work for rechecking.

Step 4. Center the required views within the work area.

Step 5. Include all dimensioning according to the most recent drafting standards.

Step 6. Add all required gear cutting specifications in the lower, right side of the paper.

Step 7. Recheck all work, and, if correct, neatly fill out the title block using light guidelines and neat lettering.

Problem 16-1

Use the following specifications to lay out a single-view drawing of a 2:1 ratio spur gear and pinion gear *in mesh*.

Spur gear:	Pinion gear:
88 teeth	hub diameter 1.625
pressure angle 20°	bore of 1.000 diameter
pitch diameter 5.500	pressure angle 20°
hub diameter 2.625	
bore of 2.000 diameter	

Use all standard drafting methods to illustrate the outside diameter, pitch diameter, and root diameter. Calculate and add the center-to-center distance between the shafts.

Problem 16-2

Draw a spur gear having the following specifications: 56 teeth, pressure angle 20°, pitch diameter 3.500, hub diameter 1.00 gear blank with overall size (width) 1.00, face size .50, bore .5625; use a ⅛" set screw.

Complete two views using the half-section method of representing gears. Do not show the gear teeth; use the conventional method of illustrating teeth. Dimension per all standard practices. Calculate and add all standard required cutting data to the lower right side of the work area. Material S.A.E. #3120 cast steel. Heat treatment: carburize .015-.020 deep.

Problem 16-3

Complete a two-view detail drawing of a pinion gear having the following specifications: calculate and add all standard cutting data: 30 teeth, pressure angle 20°, pitch diameter 6.000, hub diameter 2.000, heat treatment: carburize .050 deep, gear blank with overall size (width) 3.500, face size 1.250, bore 1.000, material S.A.E. #4620 steel.

Problem 16-4

Make a detail drawing, completely dimensioned, of a spur gear with the following specifications: diametral pitch 16, pressure angle 20°, pitch diameter 5.500, whole depth .1348, outside diameter 5.625, addendum .0625, working depth .125, circular thickness .098, chordal thickness .0633, chordal addendum .0985, whole depth .1348, and dedendum .0723, material cast brass.

Design a simple gear blank with a set screw to fasten gear to a .75 dia. shaft.

Problem 16-5

Design and draw a detail drawing, completely dimensioned, of a pair of bevel gears having teeth of a 4.0 diametral pitch; the gear with 25 teeth, the pinion gear with 13 teeth, face width of 1.00, gear shaft 1.25 diameter, and pinion shaft .875 diameter. (Calculate for FN-2 fits with the shaft.)

Design the hub diameter to be approximately twice the diameter of the shaft diameter. Backing for the gear 1.375 and for the pinion .75. Design and dimension the remaining portion to suit, using standard drafting practices. Add all cutting data to the lower right side of the work area. *Material*: S.A.E. #3120 cast steel, heat treatment: carburize .015/.020 deep.

Problem 16-6

Make a design layout of a worm and worm gear with the following specifications: shaft diameter 1.25, single-thread worm, lead .75, worm gear with 28 teeth. Add all dimensions and cutting data as required.

Problem 16-7

Sketch a clock gear chain with the following specifications:

First gear (main wheel): 84 teeth
Second gear: pinion gear 8 teeth
spur gear 60 teeth
Third gear (center shaft): pinion gear 12 teeth
spur gear 68 teeth
Fourth gear: pinion gear 7 teeth
spur gear 66 teeth
Fifth gear (escapement gear): pinion gear 7 teeth
spur gear 33 teeth

Note: In actual practice, a clock's first gear (main wheel) is usually driven by a spring. The minute hand is connected to the shaft of the third gear (center shaft) and the fifth gear is the escapement gear.

Problem 16-8

Using the sketch of a clock gear chain in Problem 16-7, answer the following questions:

1. How many times will the second gear rotate for one full revolution of the first gear?
2. How many turns will the third gear rotate for one full revolution of the second gear?
3. How many teeth of the main wheel are required to turn the third gear shaft one complete revolution? (On a clock, one revolution of the third gear equals one hour. This wheel turns 24 times for one day.)
4. What is the required revolution fo the first gear (main wheel) for one day's running time (24 turns of the center wheel).
5. What is the value of the complete gear train?
6. As this is a clock gear chain in revolutions, how many will the fifth gear (escapement) make in twelve hours? (The third gear will turn 12 times.)

Problem 16-9

Design and develop a design layout two-view drawing of a gear train with the following specifications. Design a compact clustered arrangement of the gears within an 8.0 x 10.0 supporting plates area.

First shaft: 42 teeth/4.0 outside diameter-gear
Second shaft: 6 teeth/1.0 outside diameter-pinion
Third shaft: 6 teeth/.75 outside diameter-pinion
30 teeth/2.5 outside diameter-gear
Fourth shaft: 6 teeth/.75 outside diameter-pinion
30 teeth/2.0 outside diameter-gear
Shaft sizes = .375 dia/supporting plates 1.5 apart

Problem 16-10

Locate a windup clock or some other such small gear chain device and make a design layout similar to that in Problem 16-9.